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Spatial relationships between leaf area index and topographic factors in a semiarid grassland: Joint multifractal analysis

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Abstract

A considerable portion of Canada's landmass is covered by grassland ecosystems. Insight into the grassland spatial heterogeneity will not only contribute to better understanding of the scale dependent ecological processes but will also help in management and monitoring. Leaf area index (LAI) is a key structural attribute of grassland that reflects primary production. It is well-known that topography controls grassland productivity and heterogeneity but little is known which topographic index correlates best with LAI at multiple scales. In this study, we have used multifractal and joint multifractal techniques to investigate how leaf area index in a semiarid grassland is linked with topographic factors at multiple scales. The topographic indices assessed in this study were wetness index, upslope length, and relative elevation. Our results show that field LAI is significantly correlated ($P < 0.01$) with the studied topographic factors and the effect of topography on grassland primary productivity is better explained by wetness index than upslope length or relative elevation. LAI, wetness index, and upslope length are multifractally distributed whereas distribution of relative elevation is monofractal. Joint multifractal analysis shows that the relationships between LAI and topographical factors are highly scale dependent, however, LAI is weakly correlated to relative elevation. Overall, this study suggests that the effect of topography on bioproductivity should be considered at multiple scales and multifractal and joint multifractal techniques are particularly useful in elucidating multi-scale spatial patterns of grassland ecosystems.

Keywords: Grassland, joint multifractal analysis, multifractal analysis, scale .

Abbreviation and symbols: *D*, single fractal dimension; *Dq*, the generalized fractal dimension of the UM model at the moment order of *q*; $f(\alpha)$, the multifractal spectrum; $f(\alpha,\beta)$, the joint multifractal spectrum; *i*, *k*, counting indices; *L*, length of the spatial; domain (e.g. length of the transect); LAI, leaf area index; $N(\varepsilon)$, the number of segments of size ε unit; *P*, probability level; $P_i(\varepsilon)$ and $R_i(\varepsilon)$, the probability of the measures P and *R* at the *i*th segment of the size ε units; *q*, *t*, moment orders; α', the multifractality index in the UM model; α,β, local scaling indices; ε, cell or segment size; μ, partition function; $\tau(q)$, the mass scaling function.

Introduction

Approximately 5% of Canada's land area is covered by the Prairie Ecozone and the largest percentage of Canada's Prairie Ecozone is located in Saskatchewan (Gauthier and Wiken, 2003). The composition, productivity, and diversity of North American grasslands are considerably heterogeneous (Ludwig and Tongway 1995). Information on grassland spatial heterogeneity is important for management, sampling regimes, and biodiversity monitoring. Leaf area has been found to be correlated with productivity in a variety of ecosystems, including grasslands (Gholz, 1982; Waring, 1983; Webb et al., 1983). It is a key structural characteristic of grassland ecosystems because of the role of green leaves in controlling many biological and physical processes in plants. Topographic factors such as wetness index, upslope length and relative elevation are supposed to play a key role in regulating the leaf area index in grassland. In semiarid regions, water is the limiting factor for plants. Topography is one of the important factors controlling rainfall redistribution over landscape, directly and indirectly affecting soil fertility. Furthermore, it also affects the amount of solar radiation received by plants, thus affecting photosynthesis. The importance of topography in grassland productivity has been established in the literature, and yet the search for a topographic index that best represents the integrated impacts of topography is unabated. Grassland spatial patterns are scale dependent and large-scale patterns are mainly regulated by topography or climate conditions (Lobo et al., 1998; Nellis and Briggs 1989). However, these attributes and their relationships are yet to be assessed at multiple scales. Scaling, the extrapolation or translation of information across multiple scales, can be particularly challenging due to landscape variability and nonlinearity (Wu et al., 2000). Studies investigating multi-scale variability of bioproductivity were limited for a long time because of the complexity in analysis and lack of spatial statistical techniques. Spatial statistics have only been used in landscape ecology since late 1980s (Legendre and Fortin, 1989; Rossi et al., 1992). The most frequently used techniques to characterize spatial variability are geostatistics (Sarmadian et al., 2010), spectral analysis (Perfect and Caron, 2002), wavelet analysis (He et al., 2007), multifractal analysis (Kravchenko et al., 1999) etc. Although geostatistics can assess spatial dependency in a simple way, for extreme or non-normal values it is not particularly suitable (Wang et al., 2009). On the other hand, wavelet analysis provides location dependent spatial information but it is not a useful method to assess highly skewed data. Furthermore, geostatistics, spectral, and wavelet methods only use variance and covariance. Multifractal analysis utilizes a range of statistical moments thus offers a closer look at the variability features that are obvious when multiple moments are examined but inconspicuous with the second moment (Kravchenko et al., 1999). Multifractal formalism, first proposed by Mandelbrot (1982), is suitable for variables with self-similar distribution on a spatial domain (Kravchenko et al., 2000). Due to its intrinsic nature, multifractal analysis reveals more information about data heterogeneity than other spatial techniques. For instance, minute differences in the locations of high data values in a map which is not apparent in a semivariogram can be distinguished in multifractal spectra of two data sets (Kravchenko et al., 1999). In general, multifractal attributes have different probability distribution and spatial distribution than monofractal ones and this difference is imperative for simulation and mapping as it allows us to assess their underlying processes (Wang et al., 2009). Since early 1990s, power-law relationships and fractal theory have been successfully used in many ecological studies (Sugihara and May, 1990; Harte et al., 1999). Multifractal analysis has been found to be particularly useful for examining multi-scale spatial heterogeneity of rainfall (Olsson and Niemczynowicz, 1996), soil properties (Kravchenko et al., 1999), crop yield (Zeleke and Si, 2004), vegetation patterns (Scheuring and Reidi, 1994). However, it would also be interesting to understand the associations among the variables across various scales and joint multifractal technique is highly suitable for illustrating the variability and scaling in the combined distribution of two variables on a geometric support (Zeleke and Si, 2006). This approach has been successfully used to demonstrate the relationships between topographic indices and crop yield (Kravchenko et al., 2000; Zeleke and Si, 2004). The aim of this study was to analyze the multi-scale spatial heterogeneity of LAI and topographic factors and their relationships using multifractal and joint multifractal approaches.

Materials and methods

Study site

This study was carried out in the Grasslands National Park (GNP), Saskatchewan, Canada (49^0 15' N, 107^0 09' W). GNP, established in 1984, is a mixed grass prairie ecosystem. This area has a semi-arid climate, with an annual precipitation of approximately 340 mm, mostly accruing as rainfall in the growing season (May–September). The growing days in this region are short (170 days on average) and the mean annual temperature is 3.4^0 C. The growing season is often further shortened by the lack of moisture (Csillag et al., 2001). Grassland National Park mostly consists of upland, slopeland, and valley grasslands. Upland grasslands dominate the mixed grassland ecosystem in North America, thus, sampling site was located in an upland native grassland. The site was selected as it is located along a typical rolling terrain with a soil moisture ascent. The soil in this region is a nutrient poor, shallow, clay-loam brown soil, but a wide variety of soil types (borrolls, natric, orthents, psamments and aquic) are present (Csillag et al., 2001). However, needle-and-thread grass (*Stipa comata* Trin. & Rupr.), blue grama grass (*Bouteloua gracilis* (HBK) Lang. ex Steud.), June grass (*Koeleria macrantha* (Ledeb) J.A. Schultes f.), and western wheatgrass (*Agropyron smithii* Rydb.) are the dominant grass species (He et al., 2007).

Field data collection

Field data were collected at 128 quadrats (each 50 x 50 cm^2) located at 3 m intervals along a 381 m transect. The LAI, relative elevation, wetness index, upslope length, and distance were recorded at each quadrat. LAI (the projected area of vegetative parts normalized by the subtending ground area) was measured using a LiCor LAI-2000 Plant Canopy Analyzer (LI-COR Inc., Lincoln, Nebraska, USA). The LAI-2000 was shaded when measurements were being taken to minimise the impact of glazing from direct sunshine. At each plot, LAI is the average of four automatically calculated LAI values; each was the comparison result of one above-canopy reading, followed by 10 below-canopy readings within two minutes to avoid atmospheric variation. A laser theodolite (ATT Metrology Services, Inc., California, USA) was used to determine relative elevation, angle, and distance. These topographic measurements allow for accurate calculation of slope percentage and upslope length at any point along transects. The topographical attributes used in this paper are relative elevation, upslope length, and wetness index (WI). Relative elevation was defined as the distance a point sits above or below the elevation of a reference point. Upslope length was estimated as the distance from the measurement point in the landscape to the highest relative elevation point along the slope (Si and Farrell 2004). The wetness index was calculated as the natural logarithm of the quotient of upslope length and local slope at a point.

Overview of multifractal and joint multifractal analyses

Multifractal technique can be used to characterize the scaling property of a variable measured along a transect as a mass distribution of a statistical measure on a spatial domain of the studied field (Zeleke and Si, 2004). To do this, it divides the transect into a number of self-similar segments. It identifies the differences among the subsets by using a wide range of statistical moments (Zeleke and Si, 2006). A great advantage of this technique is that it gives a much deep insight at all scales without assuming any homogeneity in the datasets or any improvised parameterization (Schertzer and Lovejoy, 1997).

The local scaling index (local variability) $\alpha(q)$ and multifractal spectrum, $f(\alpha)$ can be determined as

$$\alpha(q) = \lim_{\varepsilon \to 0} (\ln(\frac{\varepsilon}{L}))^{-1} \sum_i \mu_i(q,\varepsilon) \ln P_i(\varepsilon) \tag{1}$$

$$f(\alpha) = \lim_{\varepsilon \to 0} \left(\ln\left(\frac{\varepsilon}{L} \right) \right)^{-1} \sum_i \mu_i(q,\varepsilon) \ln \mu_i(q,\varepsilon) \tag{2}$$

where $N(\varepsilon)$ is the length of the geometric support and P is the probability function.

The partition function of moment order q can be estimated as

$$\mu(q,\varepsilon) \propto \left(\frac{\varepsilon}{L}\right)^{\tau(q)} \quad (3)$$

where ε is the scaling region i.e. the segments that follow power law. It should be noted that the variable q determines the sensitivity of the equations. The mass or correlation exponent function is related to the singularity strength.

The multifractal spectrum can be defined as (Chhabra and Jensen, 1989)

$$f(\alpha) = q.\alpha(q) - \tau(q) \quad (4)$$

The behaviour of a multifractal spectrum can be explained by employing a set of exponents called the generalized fractal dimensions, D_q. It is also known as Re´nyi dimensions and can be calculated as

$$D_q = \frac{1}{q-1} \lim_{\varepsilon \to 0} \frac{\ln \sum_i P_i(\varepsilon)}{\ln(\varepsilon)} \quad (5)$$

The D_q value at $q = 0$ is known as the capacity dimension or the box counting dimension of the geometric support of the measure. Similarly, the D_q value at $q = 1$, D_1, is called the information dimension as it provides information about the degree of variability in the distribution of a statistical measure (Zeleke and Si, 2004). However, it should be noted that from Eq. 5, D_1 is undetermined as the denominator becomes zero. Therefore, l'Hopitals rule is used to calculate D_1 (Kravchenko et al., 1999; Zeleke and Si, 2004).

$$D_1 = \frac{1}{\ln(\varepsilon / L)} \frac{\sum_i (P_i(\varepsilon).\ln P_i(\varepsilon))}{\sum_i \mu_i(q,\varepsilon)} \quad (6)$$

The Dq value at $q = 2$ (D_2), is referred to as the correlation dimension and is mathematically related to the correlation function and measures the mean distribution density of the statistical measure. Schertzer and Lovejoy (1987) formulated a universal multifractal model by making certain reasonable assumptions about the mechanisms generating multifractals. However, the critical assumption was that the underlying generator is a random variable with an exponentiated extremal Lévy distribution. The UM model, using a small number of relevant parameters, simulates the empirical moment scaling function Eq. (2) of a cascade process (Schertzer and Lovejoy 1987). Assuming conservation of the mean value, the UM model illustrates the $\tau(q)$ function as

$$\tau(q) = \frac{C1}{\alpha'-1}(q^{\alpha'} - q) \quad \alpha' \neq 1 \quad (7.1)$$

$$\tau(q) = C1.\log(q) \quad \alpha' = 1 \quad (7.2)$$

where α' is the extent of multifractality, also known as the Lévy index. The values of α' range between 0 and 2 indicating the monofractal and log-normal cases respectively. In other words, it shows how far the variable is from a monofractal type of scaling. Here $C1$ indicates the codimension (i.e., $C1 = d-D$; d is the dimension of the observation space and D is the fractal dimension) of values less than mean (moment 1) of the variable. Codimension demonstrates the sparseness of the values. Multifractal theory analyzes the distribution of a single variable (e.g. leaf area index or topographic factors) within or along its geometric support of the studied field. However, it is also intriguing to understand the joint spatial distribution of two or more measures. Joint Multifractal technique is an extension of multifractal approach for determining multi-scale spatial relationships between of two more variables. In the multifractal analysis for a single variable, the length of the transect is divided into number of smaller segments of size ε and define the probability of the measure in the ith segment of the first variable as $P_i(\varepsilon)$ and the second variable as $R_i(\varepsilon)$.The local singularity strengths corresponding to these variables can be determined as Meneveau et al. (1990).

$$P_i(\varepsilon) \propto \left(\frac{\varepsilon}{L}\right)^{\alpha} \quad (8)$$

and

$$R_i(\varepsilon) \propto \left(\frac{\varepsilon}{L}\right)^{\beta} \quad (9)$$

where α and β are the local singularity strengths or Hölder exponents corresponding to $Pi(\varepsilon)$ and $Ri(\varepsilon)$, respectively. Now the joint distributions of α and β, and the dimensions of the set resulting from the intersection of segments with iso-α and iso-β values are needed for identifying the scaling property of one variable with respect to the other. If we let $N\varepsilon(\alpha,\beta)d\alpha d\beta$ denote the number of segments of size ε with α values in the range α±dα and β values in the range $\beta \pm d\beta$, then the dimension $f(\alpha,\beta)$ of the set resulting from the intersection of segments with iso-α and iso-β values, can be determined as (Meneveau et al., 1990).

$$N_{\varepsilon}(\alpha,\beta) d\alpha d\beta \propto \left(\frac{\varepsilon}{L}\right)^{-f(\alpha,\beta)} d\alpha d\beta \quad (9)$$

$f(\alpha,\beta)$ is therefore the multifractal spectra of the joint distributions of the two variables considered. A direct method can be used for obtaining $f(\alpha,\beta)$ using the method of μ-weighted averaging (Chhabra et al., 1989; Meneveau et al., 1990). Extending the single multifractal analyses theory to the joint distributions of two variables, the partition function (the normalized μ-measures) for the joint distributions of $Pi(\varepsilon)$ and $Ri(\varepsilon)$, weighted by the real numbers q^1 and q^2 can be obtained by

$$\mu_i(q,t,\varepsilon) = \frac{P_i(\varepsilon)^q . R_i(\varepsilon)^t}{\sum_{j=1}^{N(\varepsilon)} \left[P_j(\varepsilon)^q . R_j(\varepsilon)^t\right]} \quad (10)$$

The average value of α = ln[Pi(ε)]/ln[ε/L] with respect to the μ-measures is calculated by

$$\alpha(q,t) = -\left[\ln(N(\varepsilon))\right]^{-1} \sum_{i=1}^{N(\varepsilon)} \left[\mu_i(q,t,\varepsilon).\ln(P_i(\varepsilon))\right] \quad (11)$$

whereas the average value of β = $\ln[R_i(\varepsilon)]/\ln(\varepsilon/L)$ with respect to this μ measures is determined by

$$\beta_i(q,t) = -\left[\ln(N(\varepsilon))\right]^{-1} \sum_{i=1}^{N(\varepsilon)} \left[\mu_i(q,t,\varepsilon).\ln(R_i(\varepsilon))\right] \quad (12)$$

Thus, the dimension (i.e., $f(\alpha,\beta)$) of the set on which $\alpha(q,t)$ and $\beta(q,t)$ are the mean local exponents of both measures is given by

$$f(\alpha,\beta) = -\left[\ln(N(\varepsilon))\right]^{-1} \sum_{i=1}^{N(\varepsilon)} \left[\mu_i(q,t,\varepsilon).\ln(\mu_i(q,t,\varepsilon))\right] \quad (13)$$

If q or t is set to zero, the joint partition function explained in Eq. 10 reduces to the partition function of a single measure, and therefore the joint multifractal spectrum defined by Eq. 13 becomes a single measure spectrum. However, if both q and t are set to zero, the maximum $f(\alpha,\beta)$ is attained, which is

Table 1. Statistics of biophysical property LAI and topographic factors obtained at 128 locations along the sampling transect

Variable	Minimum	Maximum	Mean	Median	SD##	R^2 #**
Leaf area index (LAI)	0.2	2.42	1.09	1.035	0.47	
Wetness Index	0.83	10.27	4.48	0.99	1.84	0.37
Relative elevation (m)	0	3.93	1.12	0.76	0.95	0.22
Upslope length (m)	0	75	28.43	19.5	18.93	0.33

Standard deviation, # Coefficient of determination between LAI and topographic factors, ** Significant at P<0.001

Table 2. Scaling nature of the variables studied by calculating selected parameters

Variable	D_0	D_1	D_2	SSR	α_{max}- α_{min}	$f(\alpha_{max})-f(\alpha_{min})$
Leaf area index	1.00	0.978	0.959	350.847 (SS)	0.573	0.35
Wetness index	1.00	0.978	0.976	370.685 (SS)	0.581	0.35
Upslope length (m)	1.00	0.943	0.913	1.327×10^3 (SS)	0.924	0.061
Relative elevation (m)	1.00	0.999	0.997	3.277 (NS)	0.081	0.228

D_0- Capacity dimension, D_1- information dimension, D_2- correlation dimension , SSR- sum of square of residuals between simulated monofractal scaling and the observed data, α_{max} - α_{min} - difference between the maximum (α at q= 10) and the minimum (α at q = -10) local fractal dimensions coarse HÖlder exponents,NS- non-significant difference (P<0.001) , SS- statistically significant difference (P<0.001)

equal to the box dimension of the geometric support of the measures. Thus, different pairs of α and β are scanned by varying the parameters q and t. Because high q or t magnifies large values and negative q or t magnifies small values, by selecting different values of q or t, we can examine the distribution of high or low values (different intensity levels) of one variable with respect to different intensity levels of the other variable. Pearson correlation analysis was used to quantitatively illustrate the variation of the scaling exponents of one variable with respect to another variable across similar moment orders. Because $f(\alpha,\beta)$ represents the frequency of the occurrence of a certain value of α and a certain value of β, high values of $f(\alpha,\beta)$ signify a strong association between the value of α and the value of β. By permuting q and t, we can examine the association of similar values (highs vs. highs or lows vs. lows) of α and β as well as dissimilar values (highs vs. lows) of α and β. Analyses were done using programs written in Mathcad 14 (Parametric Technology Corporation, Cambridge, MA).

Results

Single scale statistical analysis

Three main depressions were found at 87, 210, and 360 m and a small depression centred at 270 m along the transect (Fig 1). A coarse trend can be seen in the distribution of LAI, whereas, the upslope length and wetness index show a similar trend. LAI and topographic factors exhibit large values in the depressions and small values on the knolls. Thus, spatial variations in LAI and topography indices are evidently non-stationary, showing localized features and trends along the transect. LAI is significantly correlated with the topographic parameters, viz. relative elevation, upslope length, wetness index (Table 1). The strongest correlation was found between LAI and the wetness index (R^2 = 0.37, P < 0.001).

Multifractal analysis

The distribution of a statistical measure is considered as fractal (mono- or multi-fractal) when the moments obey power laws (Evertsz and Mandelbrot, 1992). All the attributes tested here followed power laws (Fig 2). The scaling properties can be assessed further by determining if it is simple (monofractal) or multiple (multifractal) scaling types. Sum of square difference of the residuals (SSR) and certain indicator parameters were calculated from the Chi-square goodness-of-fit test and generalized dimension function, Dq (Eq. 7) respectively (Table 2). For a distribution with a simple scaling (monofractal tendency), values of D_1 and D_2 become similar to the capacity dimension, D_0 (D_0 = 1 for a one-dimensional spatial series). When the distribution shows a tendency of multifractal type of scaling, we will see D_0>D_1> D_2. The values of D_0, D_1 and D_2 for all the studied parameters possess this property. The value of D_1 is also an excellent indicator of the degree of variability in spatial distribution of a measure. The closer the D_1 value to the capacity dimension (D_0), the more homogeneous is the distribution of the measure (Zeleke and Si, 2004). Thus, the spatial distribution of upslope length is more homogenous than those of LAI, wetness index and relative elevation. SSR values of LAI, wetness index and upslope length show a significant (P < 0.001) deviation from the simulated monofractal type scaling, whereas the SSR of relative elevation is insignificant and small. The slope of the $\tau(q)$ curves for q<0 were different from that of q>0 for LAI, wetness index and upslope length which suggests low and high density regions of the variable scale differently (Fig. 3a). In comparison with the simulated single scaling distribution, the degree of variability increases in the order of upslope length, wetness index and LAI. LAI and wetness index appeared on the same line suggesting their similar distribution patterns. The $\tau(q)$ curve of relative elevation is a straight line and overlaps the simulated monofractal type distribution indicating a single scaling property. This is in agreement with observations made from the generalized dimension analysis and the SSR values. There is a clear difference among the variables in terms of Dq values at all studied moment orders indicating highly dissimilar scaling properties of the variables (Fig. 3b). Except for certain q values (q = -10 to 0), a distinct similarity between LAI and wetness index can be seen here. The multifractality of the variables can also be evaluated by the relationship between overall fractal dimensionality (Dq) and q moment values. The Dq of a multifractal parameter changes with q whereas the Dq of a monofractal parameter remains unchanged. All the variables, except relative elevation, show a significant change which reconfirms their multifractal nature. Multifractal spectrum not only shows the resemblance and/or dissimilarity among the scaling properties of the statistical measures but also allows us to study the local scaling property of the individual parameters. Local scaling property can be defined as the spatial nature (low/high variability) of an attribute at a particular location that changes as the scale increases or

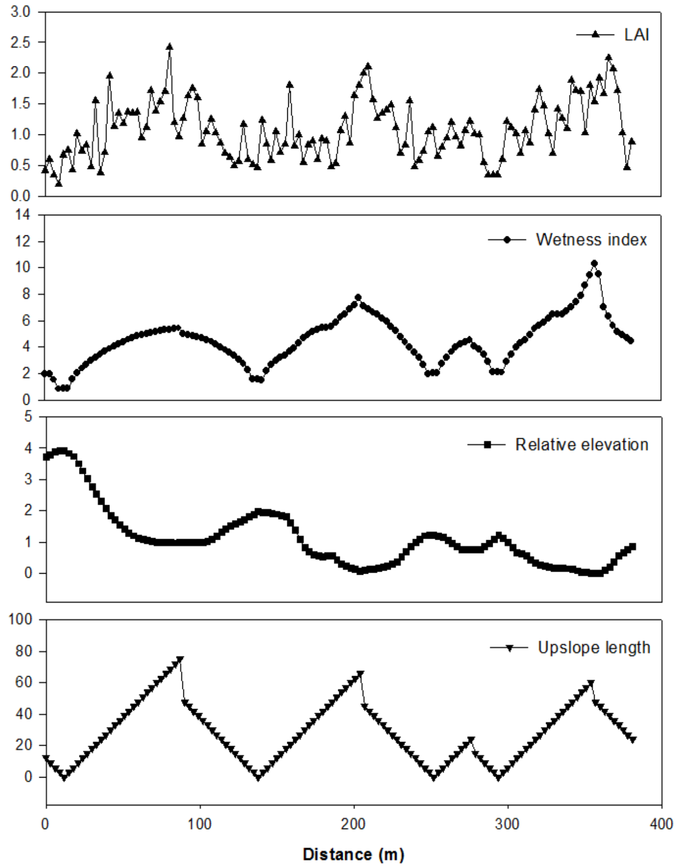

Fig 1. Grassland leaf area index (LAI) and topographic factors-relative elevation, wetness index and upslope length in 128 plots as a function of distance along the centre transect

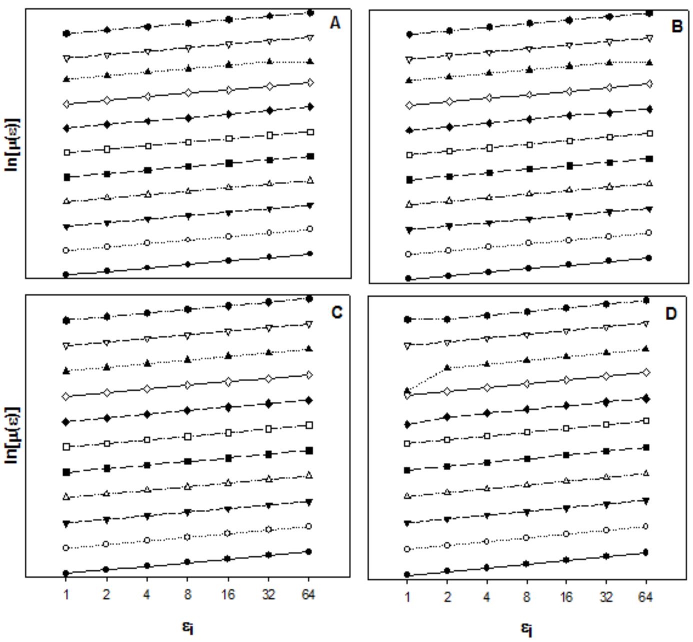

Fig 2. Natural logarithms of the normalized partition function of LAI and the topographic factors plotted against natural logarithms of the measurement scales. Clockwise from the upper left graph-LAI, wetness index, upslope length and relative elevation.

decreases. A multifractal soil attribute will have different local fractal properties (i.e. different type of singularities) whereas monofractal ones will have the same local fractal properties and multifractal spectrum (*f(α)*) reduced to one point (Wang et al., 2009). The width of the spectrum (i.e., high $\alpha_{max}-\alpha_{min}$ value) suggests the variability in the local scaling indices of the variable (Zeleke and Si, 2006). Whereas the height of the spectrum, *f(q)* corresponds to the dimension of these scaling indices. Note that the small f(q) values refer to rare events (extreme values in the distribution) whereas the largest value is the capacity dimension that is obtained at *q*=0. The distribution of upslope length has the widest spectrum {($\alpha_{max}-\alpha_{min}$) = 0.924} followed by wetness index {($\alpha_{max}-\alpha_{min}$) = 0.581} and LAI {($\alpha_{max}-\alpha_{min}$) = 0.573} (Fig. 4). The narrowest spectrum {($\alpha_{max}-\alpha_{min}$) = 0.081)} belongs to relative elevation.

Joint multifractal analysis

Scaling property of the joint distribution of the biophysical and topographic parameters was analyzed with joint multifractal analysis to assess the above observations (Fig. 5). On each plot the contour lines represent the joint dimensions, *f(α,β)* of the pair of the variables. The bottom left part of the contours exhibits the joint dimension of the high data values of the two variables, while the top right part represents the low data values (Zeleke and Si, 2006). The diagonal contours with low stretch indicate strong correlation between values corresponding to the variables in the vertical and horizontal axes (Si and Kachanoski, 2000; Zeleke and Si, 2004). A strong relationship between the scaling indices (local scaling exponents) of LAI and two topographic factors wetness index (WI) and upslope length can be observed. In both plots, the contour lines were diagonal and pulled together indicating that the high and low scaling indices of LAI were associated, respectively, with the high and low scaling indices of wetness index and upslope length. This is in agreement with the correlation coefficient values calculated at single scale and multiple scales. The correlation coefficients of the scaling indices of LAI and WI, upslope length are 0.873 and 0.80 respectively (significant at $P < 0.001$). LAI and relative elevation show negative correlation ($r = -0.115$, $P < 0.001$) at multiple scales which is also consistent with the observed correlation at a single scale.

Discussion

In this study, multifractal and joint multifractal techniques were used to analyze the spatial relationships between LAI and topographic parameters. Our results demonstrate strong positive correlations between LAI and wetness index and upslope length and a negative correlation between LAI and relative elevation. LAI is a key indicator of the primary productivity of the grassland ecosystem. Thus, the significant correlation among LAI and topographic factors implies that grassland productivity largely depends on the topography and this is in agreement with a previous study (He et al., 2007). Sellers et al. (1997) found that soil water content largely depends on the amount of precipitation accumulation, redistribution, and runoff. Thus, the strong associations between leaf area index and wetness index and upslope length found in this study are expected. Wetness index reflects the steepness of the slope as it is the ratio of the upslope length and the local terrain at a particular point. Accumulation of snow and snowmelt water at a point mainly depends on local slope. Thus, wetness index demonstrates the water storage in a location more strongly than other indicat-

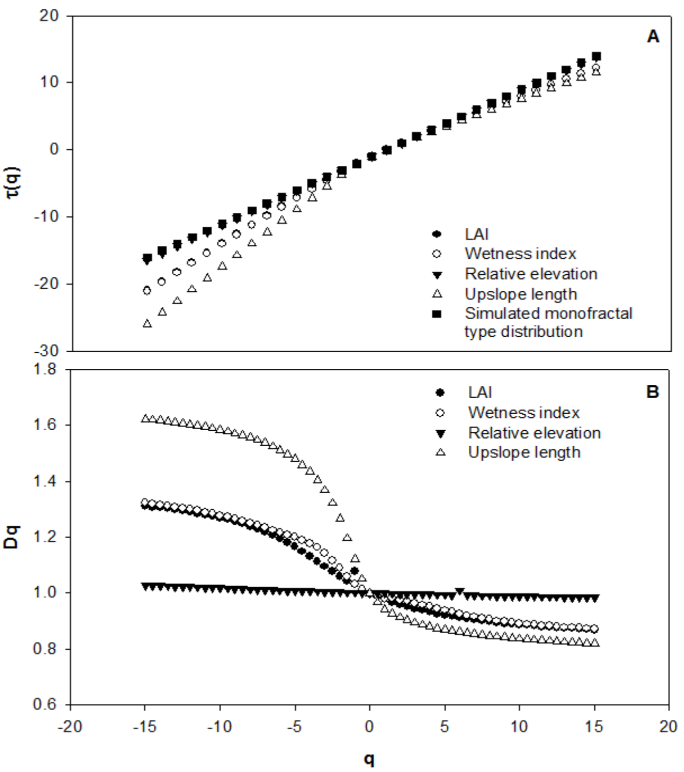

Fig 3. a) The mass exponent of the variables relative to a simulated (solid black rectangle) monofractal distribution(q = -15 to +15) using UM model of Schertzer and Lovejoy (1987). b) The generalized dimensions (Dq) the studied variables(q = -15 to +15 at 0.5 increments), field leaf area index, wetness index, relative elevation and upslope length.

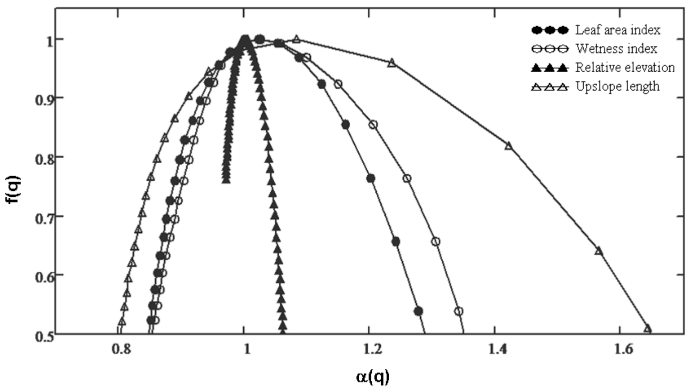

Fig 4. The multifractal spectra of the studied variables (q = -15 to +15 at 0.5 increments), field leaf area index, wetness index, relative elevation and upslope length.

ors. Soil water regime plays a significant role in soil respiration (Davidson et al., 1998), decomposition and mineralization rates (Rodriguez-Iturbe et al., 1999) and the nutrient-uptake rate of plants in semi-arid environments, which in turn determine the diversity of grassland vegetation. Topography affects soil organic matter distribution which is the key environmental factor regulating grassland vegetation at the landscape scale (Swanson et al., 1988). Topographic factors also influence the absorption and reflectance or emission of radiation by the surface, which regulate photosynthesis of plants. Thus, topography plays a major role in controlling soil water content, solar radiation, and soil organic matter content in grassland ecosystems which in turn contribute to the spatial heterogeneity of LAI. Joint multifractal analysis showed strong associations between the scaling indices of LAI and wetness index, upslope length which indicates that the relationships between these variables are valid across all spatial scales and that the spatial heterogeneity in one variable is well-reflected in the variability of the other. Hence, the relationships between biophysical properties and topographic indices should be considered at multiple spatial scales. Traditional statistical techniques can only explain the relationships at a fixed scale. Wavelet method used by He et al. (2007) found spatial correlation between LAI and topography at certain scales and thus failed to substantiate these associations across all scales. Estimation or mapping performed on the basis of single scale information derived from traditional statistical techniques may be biased. The joint multifractal analysis used in this study can be particularly useful for grassland managers and researchers to investigate correlations at all spatial scales. Leaf area index can be viewed as a surrogate of ecological functioning and net turnover of grassland ecosystem. Power–law associations of LAI in grasslands are the key features of self-similarity or fractal phenomena of biophysical properties.. Multifractal analysis based on the power-law distribution provides a framework for upscaling or downscaling. Though it is common knowledge that variability of one variable or relationships between two variables are scale-dependent and there are many techniques available for identifying that; multifractal analysis explicitly gives relationships between statistical moments and the scale, thus provides a tool for predicting the statistical moments (such as variance) at the scale of interest. Biophysical variables can be easily measured at small scales (plot scales) or large scales (through remote sensing). However, the medium scale information is hard to obtain but relevant to management and monitoring. Therefore, some sort of upscaling (from plot scale) or downscaling (from large scale) is needed and the power law scaling (multifractal) makes it possible. For example, if we know the variance of LAI at 6 m for leaf area index, we would know the variance at 12 m given the power-law distribution. Correlation and regression analyses examine at the measurement scale (sample size) how two variables are related. There is positive Pearson correlation between two variables if two variables have high values at a location. Conversely, there is a negative correlation between two variables if one variable has a high value and other variable tends to have a low value at a location. For joint multifractal, there will be a strong joint exponent (f(α,β)) if two variables are both highly intermittent (variable) at a particular location, although there may be a weak Pearson correlation. The strong joint exponent may suggest that the two variables have the same underlying controls. On the other hand, there will be a weak joint exponent if one variable is highly variable and other is very smooth around a location, which means that the two variables do not share the same control. Therefore, joint multifractal and linear correlation analyses focus on different aspects of spatial data. For management and monitoring purposes of grassland ecosystem, the joint multifractal analysis may be more relevant. Multifractal analysis can zoom-in extreme high and low values. In grassland ecosystems, extremely high or low turnover of a particular location may suggest difference in richness and abundance in species or patchy growth for species due to unusual nutrient and/or moisture availability. The joint multifractal analysis considers the location-specific joint variability of two variables. If an attribute is homogeneous at a particular location, the regulatory factors are most likely homogeneous. Thus, it

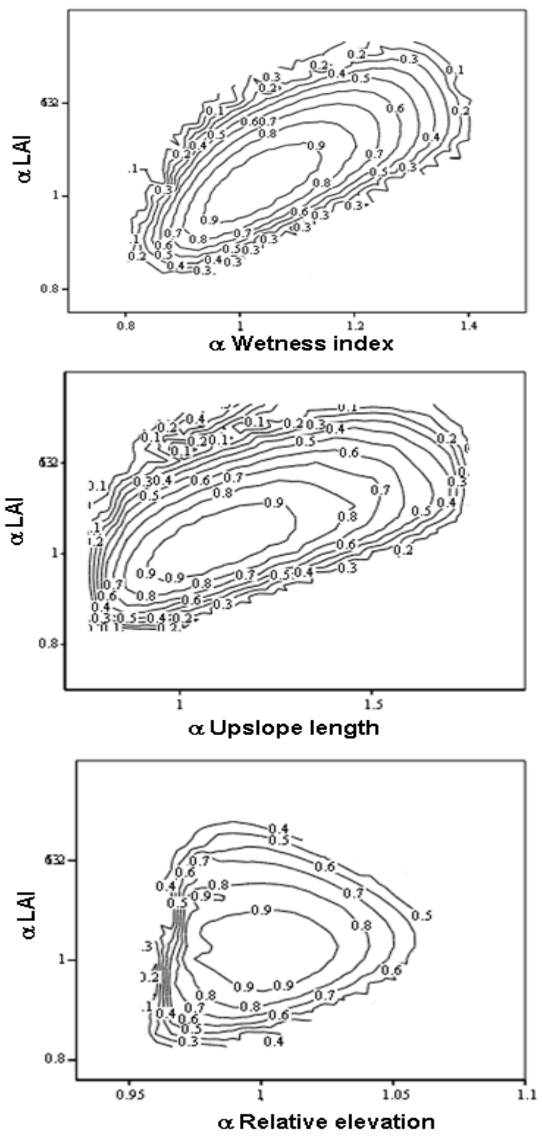

Fig 5. The multifractal spectrum of the joint distribution of field leaf area index (vertical axis) and three topographic factors- wetness index (WI), upslope length and relative elevation (horizontal axis). Contour lines show the joint distribution of the two scaling indices $\alpha(q,t)$ and $\beta(q,t)$.

highlights the importance of consistency of the topographic indices to achieve consistent productivity.

Conclusions

Leaf area index and the studied topographic factors, and their relationships are highly scale dependent and should be considered at multiple spatial scales. For monitoring and management of semiarid grassland ecosystems, the satellite images taken at a large scale may not correspond to ground measurements at a small scale. For mono-fractal soil properties (such as relative elevation), upscaling from small scale measurements to large-scale applications can be carried out at any moment order (for both high and low data values) with the same scaling exponent. For multifractal ones (such as LAI), however, scaling transformation can be conducted, but needs different scaling exponent for different moment order (for only high or low data values). Therefore, upscaling for LAI and wetness index would be much more demanding in data than that for elevation. This becomes particularly important when matching of ground truth and satellite image data is needed.

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